This relationship from the previous theorem along with the Newton interpolatory divided-difference formula gives us an alternate method for formulating Hermite approximations.

Suppose we are given distinct numbers $x_0, x_1, \ldots, x_n$ along with the values of f and f' at these values. Define a new sequence $z_0, z_1, \ldots, z_{2n+1}$ by

$$z_{2k} = z_{2k+1} = x_k, \quad \text{for each } k = 0, 1, \ldots, n.$$

Then construct a divided-difference table using the variables $z_0, z_1, \ldots, z_{2n+1}$. However, since $z_{2k} = z_{2k+1} = x_k$ for each k, we cannot use

$$f[z_{2k}, z_{2k+1}] = \frac{f[z_{2k+1}] - f[z_{2k}]}{z_{2k+1} - z_{2k}}$$

since the denominator is 0. But for each k, $f[x_k, x_{k+1}] = f'(\xi_k)$ for some number ξ_k in (x_k, x_{k+1}) by the MVT and $\lim_{x_{k+1} \to x_k} f[x_k, x_{k+1}] = f'(x_k)$. Thus we use the substitutions $f[z_{2k}, z_{2k+1}] = f'(x_k)$ and so we use

$$f'(x_0), f'(x_1), \ldots, f'(x_n)$$

for

$$f[z_0, z_1], f[z_2, z_3], \ldots, f[z_{2n}, z_{2n+1}].$$

Everything else in the divided-difference table is done as usual.

Theorem (Divided-Difference Form of the Hermite Polynomial). *If $f \in C^1[a, b]$ and $x_0, x_1, \ldots, x_n$ are distinct in $[a, b]$, then*

$$H_{2n+1}(x) = f[z_0] + \sum_{k=1}^{2n+1} f[z_0, z_1, \ldots, z_k] \underbrace{(x - z_0) \cdots (x - z_{k-1})}_{basis functions},$$

where $z_{2k} = z_{2k+1} = x_k$ and $f[z_{2k}, z_{2k+1}] = f'(x_k)$ for each $k = 0, 1, \ldots, n$.