6. Parametric Curves

Parametric curves are needed for generating curves where one variable is not a function of the other such as the curve in figure 3.12 on page 103.

To determine a polynomial or piecewise polynomial to connect

$$(x_0, y_0), (x_1, y_1), \ldots, (x_n, y_n)$$

in order, we use a parameter t on an interval $[t_0, t_n]$ with $t_0 < t_1 < \cdots, < t_n$ and find functions x and y such that

$$x_i = x(t_i) \text{ and } y_i = y(t_i) \text{ for } i = 0, 1, \ldots, n.$$

We can use any of the methods so far to find x and y.

Computer graphics require rapid generation of smooth curves that are quickly and easily modified. Computer graphics also require that changes in one portion of a curve to have little effect on other portions — this rules out interpolating polynomials and splines.

The choice is usually piecewise cubic Hermite polynomials. Each piece is completely determined by specifiying the endpoints and the derivatives at the endpoints. Each cubic polynomial, x and y, has 4 parameters, totalling 8 needed conditions.

We can simplify the process to one of determining two cubic Hermite polynomials in the parameter t, where $t_0 = 0$, $t_1 = 1$, given the endpoint data $(x(0), y(0))$, and $(x(1), y(1))$ and the derivatives $\dfrac{dy}{dx}$ at $t = 0$ and $t = 1$.