Problem (Page 136 #5). Use the following data to approximate $\int_1^5 f(x)\,dx$ as accurately as possible.

x	1	2	3	4	5
$f(x)$	2.4142	2.6734	2.8974	3.0976	3.2804

We begin by using the trapezoidal rule for $n = 1, 2, 4$ subintervals, so $n = 3$. then $m_1 = 1$, $m_2 = 2$, and $m_3 = 4$. With $b - a = 5 - 1 = 4$, $h_1 = \frac{4}{1} = 4$, $h_2 = \frac{4}{2} = 2$, and $h_3 = \frac{4}{4} = 1$.

$$R_{1,1} = \frac{h_1}{2}\Big[f(1) + f(5)\Big] = \frac{4}{2}\Big[2.4142 + 3.2804\Big] = 11.3892$$

$$R_{2,1} = \frac{1}{2}\Big[R_{1,1} + h_1 f(3)\Big] = \frac{1}{2}\Big[11.3892 + 4(2.8974)\Big] = 11.4894$$

$$R_{3,1} = \frac{1}{2}\Big[R_{2,1} + h_2(f(2) + f(4))\Big] = \frac{1}{2}\Big[11.5228 + 2(2.6734 + 3.0976)\Big] = 11.5157$$

$$R_{2,2} = \frac{4R_{2,1} - R_{1,1}}{4 - 1} = \frac{4(11.4894) - 11.3892}{4 - 1} = 11.5228$$

$$R_{3,2} = \frac{4R_{3,1} - R_{2,1}}{4 - 1} = \frac{4(11.5157) - 11.4894}{4 - 1} = 11.5244666667$$

$$R_{3,3} = \frac{16R_{3,2} - R_{2,2}}{16 - 1} = \frac{16(11.5244666667) - 11.5228}{16 - 1} = 11.5245777778$$

Placing these numbers in our table yields

$O(h_k^2)$	$O(h_k^4)$	$O(h_k^6)$
11.3892		
11.4894	11.5228	
11.5157	11.5244666667	11.5245777778

Thus $\int_1^5 f(x)\,dx \approx 11.5246$ to the four places of the original data.