CHAPTER 1

Preliminaries

Why non-pictured analysis?

f is continuous at x if $\lim_{h \to 0} f(x+h) = f(x)$

and

f is differentiable at x if $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ exists.

Then

Differentiability $\Longrightarrow$ continuity,

but

continuity $\not\Longleftarrow$ differentiability.

In 1854, Karl Weierstrauss gave an example of a continuous function which was nowhere differentiable:

$$F(x) = \sum_{n=0}^{\infty} \frac{\cos(3^n x)}{2^n}.$$

Then term by term differentiation gives

$$\sum_{n=0}^{\infty} \left(\frac{3}{2}\right)^n \sin(3^n x),$$

which diverges when x is not a multiple of π.