$(MI \Longrightarrow SI)$

We are given that MI is true.

Let $S \subseteq \mathbb{N} \ni 1 \in S$ and $(\{1, 2, \ldots, k\} \subseteq S \Longrightarrow k+1 \in S)$.

[To show $S = \mathbb{N}$ by using MI.]

Suppose $k \in S$. Now $1 \in S \Longrightarrow \{1\} \subseteq S \Longrightarrow 2 \in S \Longrightarrow$

$\{1, 2\} \subseteq S \Longrightarrow 3 \in S \Longrightarrow \{1, 2, 3\} \subseteq S$. Continuing, within a finite number of steps, we get $\{1, 2, \ldots, k\} \subseteq S \Longrightarrow k \in S \Longrightarrow k+1 \in S$ (by MI).

Thus, by MI, $S = \mathbb{N}$ as SI is proved.

$(SI \Longrightarrow MI)$

We are given that SI is true.

Let $S \subseteq \mathbb{N} \ni 1 \in S$ and $(k \in S \Longrightarrow k+1 \in S)$.

[Tos show $S = \mathbb{N}$ by using SI.] Suppose $\{1, 2, \ldots, k\} \in S$. Then $k \in S \Longrightarrow k+1 \in S$.

Thus $S = \mathbb{N}$ by SI, so MI is proved. $\square$

As a result of this equivalence, once one of the three is taken as an axiom, the two others follow as Corollaries.