$\square$

EXAMPLE.

(3) Suppose $f : I \to \mathbb{R}$ is differentiable on I and $g(y) = y^n \ \forall\ y \in \mathbb{R}, n \in \mathbb{N}$. Then $g'(y) = ny^{n-1}$, so

$$(g \circ f)'(x) = g'\big[f(x)\big] \cdot f'(x) \quad \text{for } x \in I$$

or

$$(f^n)'(x) = n\big[f(x)\big]^{n-1} \cdot f'(x) \quad \text{for } x \in I.$$

(4) $f(x) = \begin{cases} x^2 \sin \frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$. For $x \neq 0$,

$$f'(x) = 2x \sin \frac{1}{x} + x^2\Big(\cos \frac{1}{x}\Big)\Big(-\frac{1}{x^2}\Big) = 2x \sin \frac{1}{x} - \cos \frac{1}{x}.$$

From Example (1), $f'(0) = 0$, but $\lim_{x \to 0} f'(x)$ DNE,

so f' is not continuous at $0 \Longrightarrow$

$f''(0)$ DNE. Note that $f''(x)$ exists for $x \neq 0$.