

EXAMPLE.

(1) $f(x) = k \in \mathcal{R}[a, b] \forall k \in \mathbb{R}$.

PROOF. Let $\dot{P} = \left\{ ([x_{i-1}, x_i], t_i) \right\}_{i=1}^n$ be any tagged partition of $[a, b]$. Then

$$\begin{aligned} S(f; \dot{P}) &= \sum_{i=1}^n f(t_i)(x_i - x_{i-1}) \\ &= \sum_{i=1}^n k(x_i - x_{i-1}) \\ &= k \sum_{i=1}^n (x_i - x_{i-1}) \\ &= k(b - a). \end{aligned}$$

Thus, $\forall \epsilon > 0$, choose $\delta_\epsilon = 1$. Then $\|\dot{P}\| < \delta_\epsilon \implies$

$$|S(f; \dot{P}) - k(b - a)| = 0 < \epsilon.$$

Since ϵ is arbitrary, $f \in \mathcal{R}[a, b]$ and $\int_a^b f = k(b - a)$. □