NOTE.

(1) If $E = \emptyset$, hypothesis (a) is automatically satisfied.

(2) If f is not defined at $c \in E$, take $f(c) = 0$.

(3) Even if $E = \emptyset$, hypothesis (c) is not automatically satisfied since $\exists$ functions $F \ni F' \notin \mathcal{R}[a, b]$ (see Example (4) below)

EXAMPLE.

(1) If $F(x) = \dfrac{1}{3}x^3$ on $[a, b]$, then $f(x) = F'(x) = x^2$ is continuous on $[a, b] \Longrightarrow f \in \mathcal{R}[a, b]$. With $E = \emptyset$ here,

$$\int_a^b x^2\,dx = F(b) - F(a) = \frac{1}{3}(b^3 - a^3).$$

(2) $F(x) = |x - 1|$ for $x \in [-10, 10]$.

$$f(x) = F'(x) = \begin{cases} -1, & x \in [-10, 1) \\ 1, & x \in (1, 10] \end{cases}.$$

Here $E = \{1\}$. Defining $f(1) = 0$, f is a step function $\Longrightarrow f \in \mathcal{R}[-10, 10]$. Thus

$$\int_{-10}^{10} f(x)\,dx = F(10) - F(-10) = 9 - 11 = -2.$$

(3) $F(x) = 4x^{1/4}$ for $x \in [0, b]$.

F is continuous on $[0, b]$ and $F'(x) = x^{-3/4}$ on $(0, b]$.

Since $f(x) = F'(x)$ is not bounded on $[0, b]$, $f \notin \mathcal{R}[0, b]$

and so the FTC does not apply.