COROLLARY (2.2.4). *If* $a, b \in \mathbb{R}$, *then*

$$(a) \quad \big||a| - |b|\big| \leq |a - b|$$
$$(b) \quad |a - b| \leq |a| + |b|$$

NOTE. These are also referred to as triangle inequalities.

PROOF. [We use a "smuggling" technique.]

(a)

$$|a| = |a - b + b| \leq |a - b| + |b| \Longrightarrow$$
$$|a| - |b| \leq |a - b|.$$
$$|b| = |b - a + a| \leq |b - a| + |a| \Longrightarrow$$
$$|b| - |a| \leq |b - a| \Longrightarrow$$
$$-|a - b| \leq |a| - |b|.$$

Thus

$$-|a - b| \leq |a| - |b| \leq |a - b| \Longrightarrow$$
$$\big||a| - |b|\big| \leq |a - b|$$

by Theorem 2.2.2.(c)

(b) Just replace b by $(-b)$ in the triangle inequality. □

COROLLARY (2.2.5). $\forall a_1, a_2, \ldots, a_n \in \mathbb{R}$,

$$|a_1 + a_2 + \cdots + a_n| \leq |a_1| + |a_2| + \cdots + |a_n|.$$