

COROLLARY (2.4.5). *If $t > 0$, then $\exists n_t \in \mathbb{N} \ni 0 < \frac{1}{n_t} < t$.*

PROOF. Since $\inf \left\{ \frac{1}{n} : n \in \mathbb{N} \right\} = 0$ and $t > 0$,

t is not a lower bound of $\inf \left\{ \frac{1}{n} : n \in \mathbb{N} \right\}$, so

$$\exists n_t \in \mathbb{N} \ni 0 < \frac{1}{n_t} < t. \quad \square$$

COROLLARY (2.4.6). *If $y > 0$, then $\exists n_y \in \mathbb{N} \ni n_y - 1 \leq y < n_y$.*

PROOF. Let $E_y = \{m \in \mathbb{N} : y < m\}$.

By the Archimedean property, $E_y \neq \emptyset$.

By Well-Ordering, E_y has a least element, say n_y .

Then $n_y - 1 \notin E_y$, so

$$n_y - 1 \leq y < n_y. \quad \square$$