1.2. Mthematical Induction

Well-Ordering Property (WO) of $\mathbb{N}$ – an axiom

Every non-empty subset of $\mathbb{N}$ has a least element.

$\Longleftrightarrow$

If $S \subseteq \mathbb{N}$ and $S \neq \emptyset$, $\underbrace{\exists}_{\text{there exists}}$ $m \in S$ $\underbrace{\ni}_{\text{such that}}$ $m \leq k$ $\underbrace{\forall}_{\text{for every}}$ $k \in S$.

Principle of Mathematical Induction (MI) - an axiom

Let $S \subseteq \mathbb{N} \ni$

(1) $1 \in S$;

(2) if $k \in S$, then $k + 1 \in S$.

Then $S = \mathbb{N}$.

Applied Version of MI

Let $n_0 \in \mathbb{N}$ and $P(n)$ be a statement $\forall n \geq n_0, n \in \mathbb{N}$. Suppose

(1) $P(n_0)$ is true;

(2) $\forall k \geq n_0$, "$P(k)$ true" (the induction hypothesis) $\Longrightarrow$ "$P(k+1)$ true".

Then $P(n)$ is true $\forall n \geq n_0$.