PROBLEM (Page 16 #8). Prove that $\underbrace{5^n - 4n - 1 \text{ is divisible by } 16}_{P(n)}$ $\forall n \in \mathbb{N}$.

PROOF. Let $S \subseteq \mathbb{N} \ni$ $5^n - 4n - 1$ is divisible by 16.

$1 \in S$ since $5 - 4 - 1 = 0$ is divisible by 16.

Suppose $k \in S$, i.e. $5^k - 4k - 1$ is divisible by 16 (induction Hypothesis).

Then

$$5^{k+1} - 4(k+1) - 1 =$$
$$5^{k+1} - 4k - 5 =$$
$$(5^{k+1} - 20k - 5) + 16k =$$
$$5\underbrace{(5^k - 4k - 1)}_{\text{divisible by } 16} + 16k$$

is divisible by 16, so $k + 1 \in S$.

Thus, by math induction, $S = \mathbb{N}$. $\square$