PROBLEM (Page 16 #4). Prove $2^n < n!\ \forall n \geq 4, n \in \mathbb{N}$.

PROOF. Let $P(n)$ be $(2^n < n!)$.

$P(4)$ is true since $2^4 = 16 < 24 = 4!$.

Suppose $P(k)$ is true, i.e., $2^k < k!$, $k \geq 4$.

Then

$$\begin{aligned} &2^{k+1} \\ &= 2^k * 2 \\ &< k! \cdot 2 \\ &< k!(k+1) \\ &= (k+1)!, \end{aligned}$$

so $P(k+1)$ is true.

Thus $P(n)$ is true $\forall n \geq 4, n \in \mathbb{N}$. □

$\boxed{\textit{Homework}}$ Pages 15-16#3, 6, 13.

Principal of Strong Induction (SI) - an axiom.

Let $S \subseteq \mathbb{N} \ni$

(1') $1 \in S$,

(2') if $\{1, 2, \ldots, k\} \subseteq S$, then $k + 1 \in S$.

Then $S = \mathbb{N}$.