THEOREM (1). $WO \iff MI \iff SI$.

PROOF.

$(WO \implies MI)$.

Suppose $S \subseteq \mathbb{N} \ni 1 \in S$ and $(k \in S \implies k+1 \in S)$ [so we need to prove $S = \mathbb{N}$], but $S \neq \mathbb{N}$.

Then $\mathbb{N} \backslash S \neq \emptyset$,

so by WO $\mathbb{N} \backslash S$ contains a least element, say m.

Since $1 \in S$, $m \neq 1$, so $m > 1$ and $m - 1 \in \mathbb{N}$.

$\big[$Where is $m - 1$?$\big]$

Since $m - 1 < m$ and m is the least element of $\mathbb{N} \backslash S$,

$m - 1 \in S$.

Then $(m-1)+1 = m \in S$, contradicting that $m \in \mathbb{N} \backslash S$. Since our assumption that $S \neq \mathbb{N}$ led to this contradiction, $S = \mathbb{N}$, proving MI.