$(MI \Longrightarrow WO)$

Suppose $\emptyset \neq S \subseteq \mathbb{N}$

[we need to prove S has a least element] and S does not have a least element.

Let T be the set of natural numbers that every element of S is greater than or equal to. [We use induction to show $T = \mathbb{N}$]

$1 \in T$, since if $x \in S$, $x \in \mathbb{N}$, and so $x \geq 1$.

Suppose $k \in T$ (induction hypothesis).

Then $\forall x \in S$, $x \geq k$ [so where is $k + 1$?],

so $k \notin S$ (since it would be a least element if it were in S).

Thus, $\forall x \in S$, $x > k \Longrightarrow$

$x \geq k + 1 \Longrightarrow k + 1 \in T$. Then, by MI, $T = \mathbb{N}$

Since $S \neq \emptyset$, $\exists n \in S \Longrightarrow n \in \mathbb{N} \Longrightarrow n + 1 \in \mathbb{N} = T$.

But then, since $n \in S$ and $n + 1 \in T$, $n > n + 1$, a contradiction. Since assuming S does not have a least lement led to this contradiction, S does have a least element and WO is proved.