>

A Normal Distribution

```
> restart;
```

Normal Probability Density Function

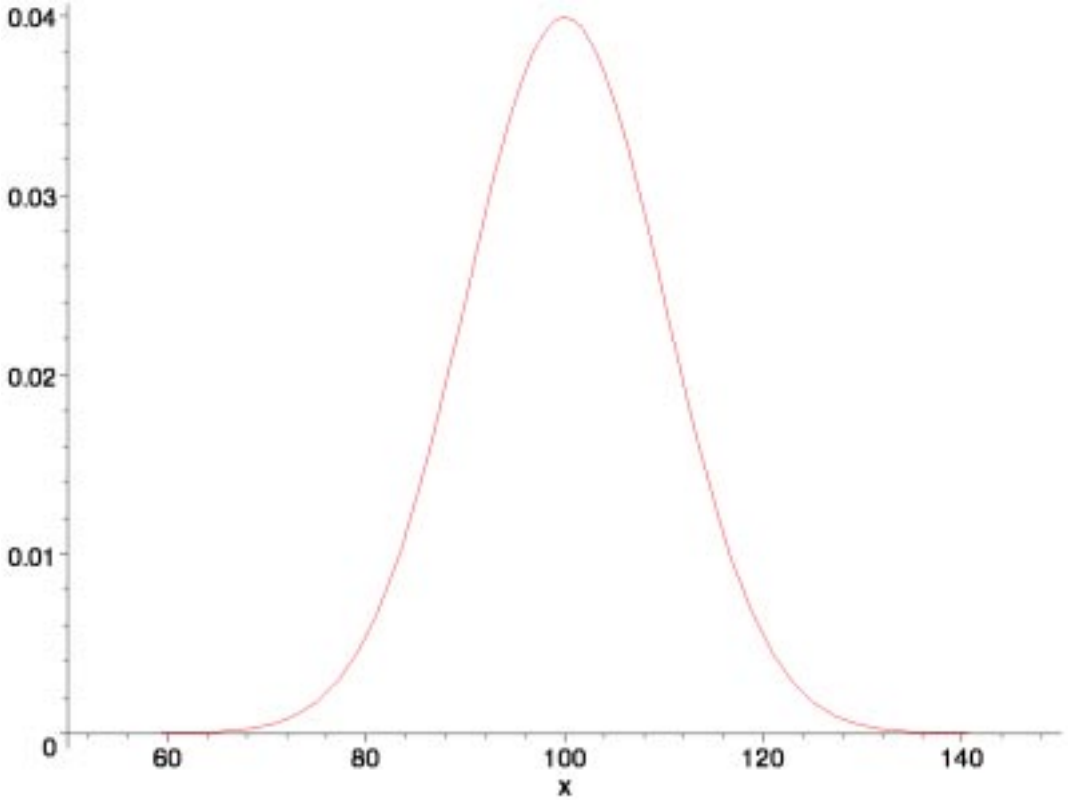
We wish to find the graph of the normal probability density function with mean $\mu = 100$ and standard deviation $\sigma = 10$. The formula for this function is $f(x) = \frac{1}{10\sqrt{2\pi}} e^{\left(-\frac{(x-100)^2}{200}\right)}$. We enter the formula easily the Maple way..

```
> f:=(1/(10*sqrt(2*Pi)))*exp(-(x-100)^2/200);
```

$$f := \frac{1}{20}\frac{\sqrt{2}\, e^{(-1/200\,(x-100)^2)}}{\sqrt{\pi}}$$

Now we plot the graph.

```
> plot(f,x=50..150);
```



The graph looks like it is 0 to the left of 60 and to the right of 140. But let's check f(200).

```
> evalf(subs(x=200,f));
```

$$.7694598625\ 10^{-23}$$

Pretty small, but still there. Now let's check f(20000).

```
> evalf(subs(x=20000,f));
```

$$.6486606185\ 10^{-859926}$$

Also still there. Actually, $f(x) > 0$ for all x. Now let's find the integral from negative to positive infinity.

```
> int(f,x=-infinity..infinity);
```

1

Next we want to find the second derivative of the function.

```
> f2:=diff(f,x$2);
```

$$f2 := -\frac{1}{2000}\frac{\sqrt{2}\ \mathbf{e}^{(-1/200\,(x-100)^2)}}{\sqrt{\pi}} + \frac{\frac{1}{20}\sqrt{2}\left(-\frac{1}{100}x+1\right)^2\mathbf{e}^{(-1/200\,(x-100)^2)}}{\sqrt{\pi}}$$

To find the x-values of the inflection points, we set the second derivative equal to 0 and solve for x.

```
> solve(f2=0,x);
```

110, 90

Notice that these two x-values of the points of inflection are exactly one standard deviation from the mean in each direction. Now let's take the integral covering one standard deviation in each direction.

```
> evalf(int(f,x=90..110));
```

.6826894920

Covering two standard deviations.

```
> evalf(int(f,x=80..120));
```

.9544997360

Covering three standard deviations.

```
> evalf(int(f,x=70..130));
```

.9973002039

```
>
```

Normal Probability Distribution (Cumulative Density) Function

Now we wish to find the graph of the normal probability distribution (cumulative density) function with mean $\mu = 100$ and standard deviation $\sigma = 10$. The formula for this function is

$F(x) = \int_{-\infty}^{x} \frac{1\ \mathbf{e}^{\left(\frac{(-(y-100))^2}{200}\right)}}{10\sqrt{2\,\pi}}\,dy$. We enter the formula.

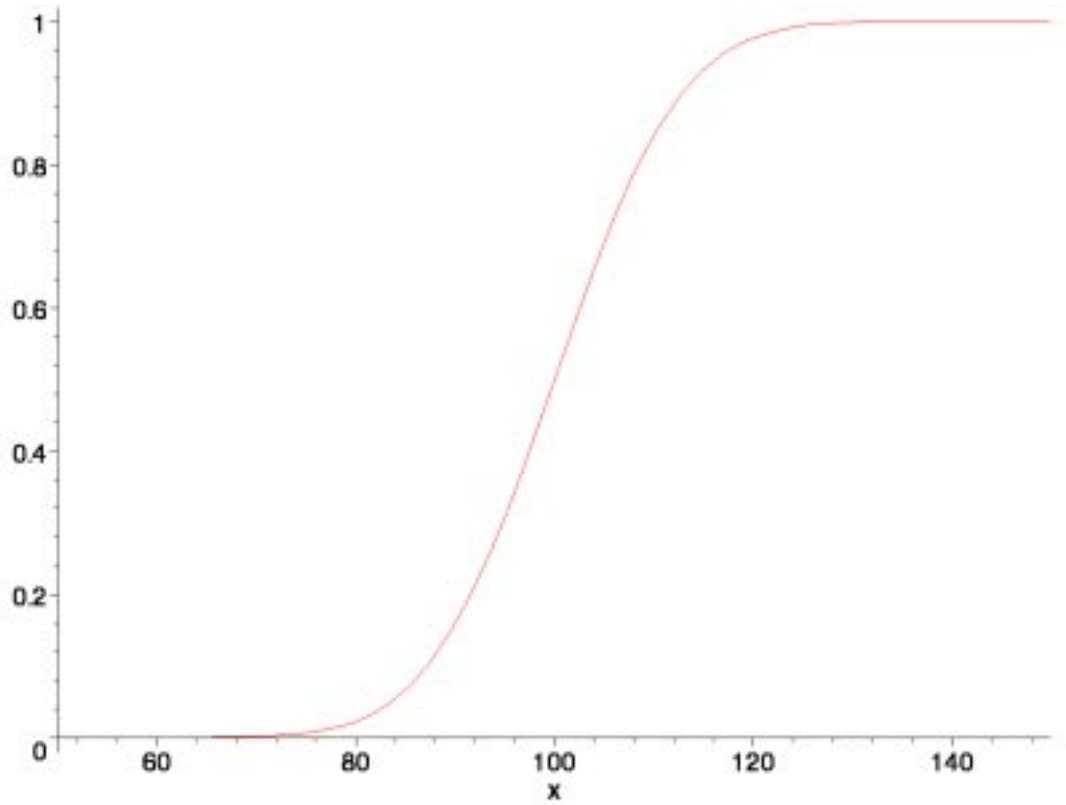
```
> restart;
> f:=(1/(10*sqrt(2*Pi)))*exp(-(y-100)^2/200);
```

$$f := \frac{1}{20}\frac{\sqrt{2}\ \mathbf{e}^{(-1/200\,(y-100)^2)}}{\sqrt{\pi}}$$

```
> F:= int(f,y=-infinity..x);
```

$$F := \frac{1}{2}\,\mathrm{erf}\left(\frac{1}{20}\sqrt{2}\ x - 5\sqrt{2}\right) + \frac{1}{2}$$

```
> plot(F,x=50..150);
```



Let's check some values.

```
> "F(-infinity)"=evalf(subs(x=-infinity,F));
```

"F(-infinity)" = 0.

```
> "F(50)"=evalf(subs(x=50,F));
```

"F(50)" = $.2866\ 10^{-6}$

```
> "F(100)"=evalf(subs(x=100,F));
```

"F(100)" = .5000000000

```
> "F(150)"=evalf(subs(x=150,F));
```

"F(150)" = .9999997134

```
> "F(infinity)"=evalf(subs(x=infinity,F));
```

"F(infinity)" = 1.000000000

```
>
```